What can it be used for

and

what can we learn from it?

Richard B. Kreckel, Johannes Gutenberg University, Mayence

Workshop on Open Source Computer Algebra Systems

Lyon, France, 21 May 2002

Overview

- The GiNaC Framework
  - Objective and Idea
  - The Class Hierarchy
- Demonstration: Selected Symbolic Capabilities
  - Manipulation of Polynomials
  - Power Series Expansion
- Symbolic Computation in C++: Relevant Design Patterns
  - Reference Counting
  - Anonymous Evaluation
  - Delegation vs. the Visitor Pattern
  - The Flyweight and the Bridge Patterns
  - Potential for Object Depletion

The GiNaC Framework: Objective and Idea

Why:

- Dissatisfaction with MapleV Rn for HEP computations:
  - Limitations in polynomial representation (fixed in 2000 with Maple6)
  - Many linguistical problems (weird scope)
  - Support by manufacturer? Patch releases? What's that?
- Situation in 1998: Nearly everything was closed-source or under 'strange' licenses (Form, Pari, NTL, Singular. . .) (different now!)
- C++ is *lingua franca* for us physicists
- Wanted a testbed for symbolic manipulation
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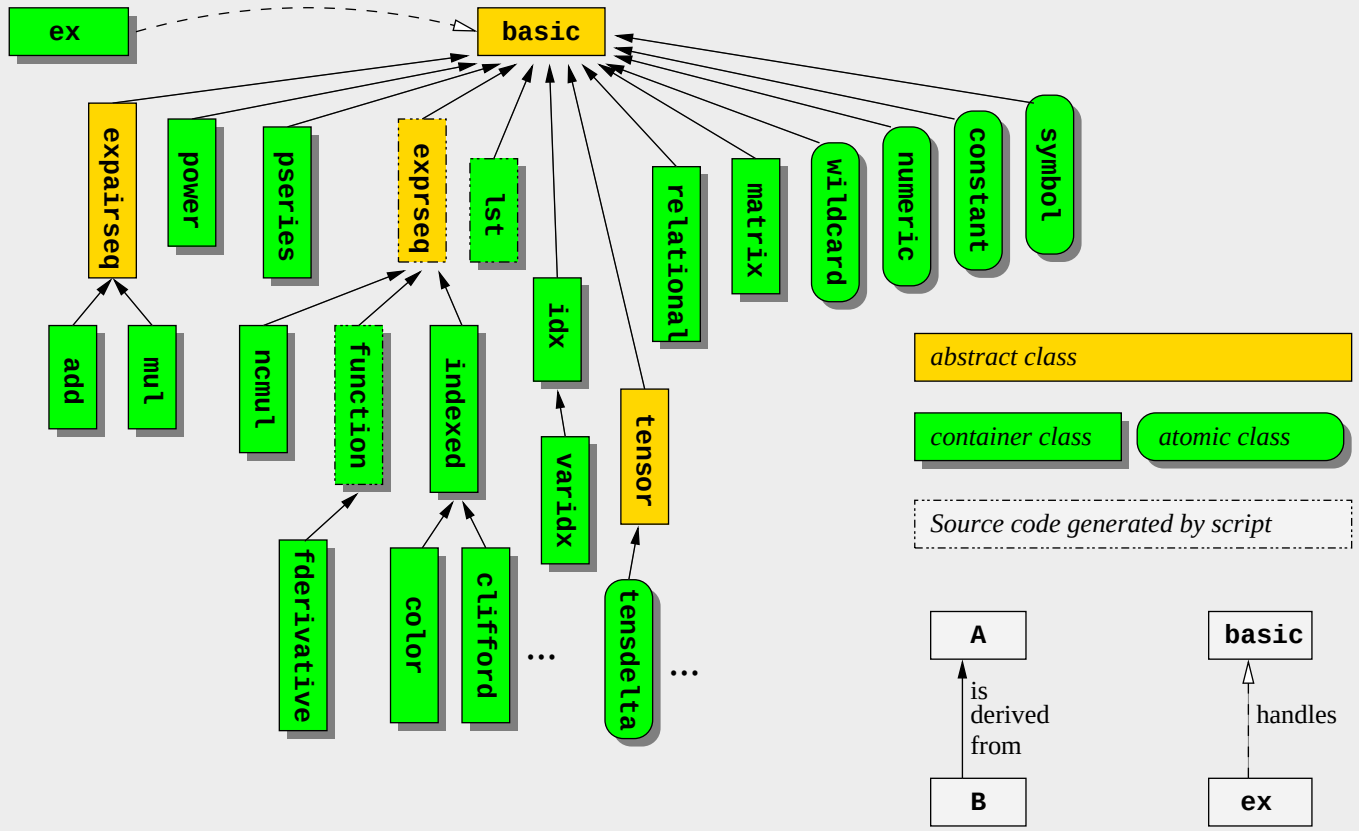
GiNaC: (Ch. Bauer, A. Frink, R. Kreckel)

- Absolutely no limits (arithmetic, size of expressions)
- Free license (GPL), open development model (CVS, patches welcome)
- Not yet another language $\Rightarrow$ program in C++ (ISO/IEC 14882)
- Provides all that is needed for HEP computations:
  - Arbitrary precision arithmetic (through CLN)
  - Multivariate polynomial GCDs, though no factorization
  - Powerful series expansion
  - Noncommutative objects: SU(2), SU(3), etc. . .

The GiNaC Framework: The Class Hierarchy



- ▷ Memory management: Reference counting
- ▷ Objects are hashed, for fast comparison (Fibonacci hash)
- ▷ Class `numeric` wraps Bruno Haible's CLN (GPL, ∼100kloc)
  C++ library, arbitrary precision arithmetic, type retraction, refcounted
- ▷ Algebraic notation through operator overloading ⇒ intuitive syntax

Demonstration: Selected Symbolic Capabilities

Special Relativity (Einstein, 1905): mass m is a function of velocity v

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```
1  #include <iostream>
2  #include <ginac/ginac.h>
3  using namespace std;
4  using namespace GiNaC;
5
6  int main(void)
7  {
8      const symbol v("v"), c("c");
9
10     ex gamma        = 1 / sqrt(1 - pow(v/c, 2));
11     ex mass_nonrel  = gamma.series(v == 0, 20);
12
13     cout << "the relativistic mass increase with v is\n"
14          << mass_nonrel << endl;
15
16     cout << "the inverse square of this series is\n"
17          << pow(mass_nonrel, -2).series(v == 0, 123) << endl;
18
19     return 0;
20 }
```

Symbolic Computation in C++: Relevant Design Patterns

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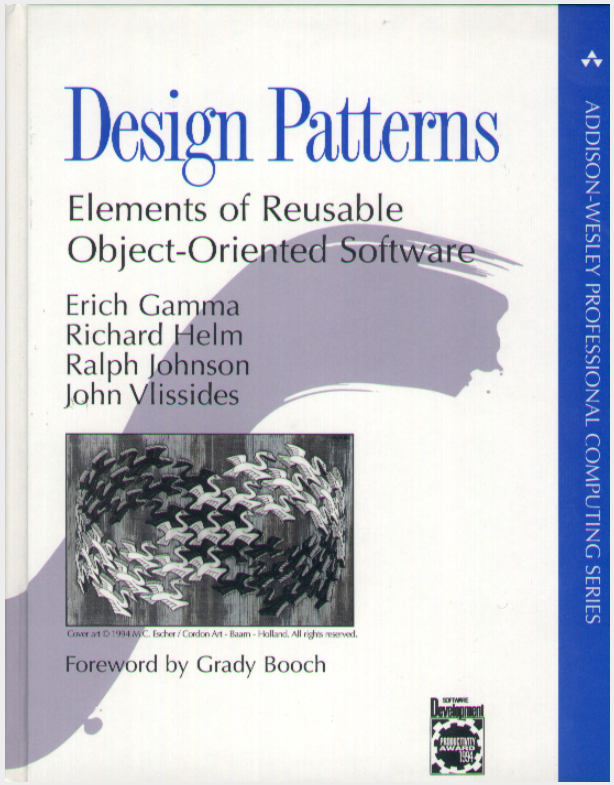

What is a Design Pattern?

"*Each pattern describes a problem which occurs over and over again in our environment, and then describes the core of the solution to that problem, in such a way that you can use this solution a million times over, without ever doing it the same way twice.*"

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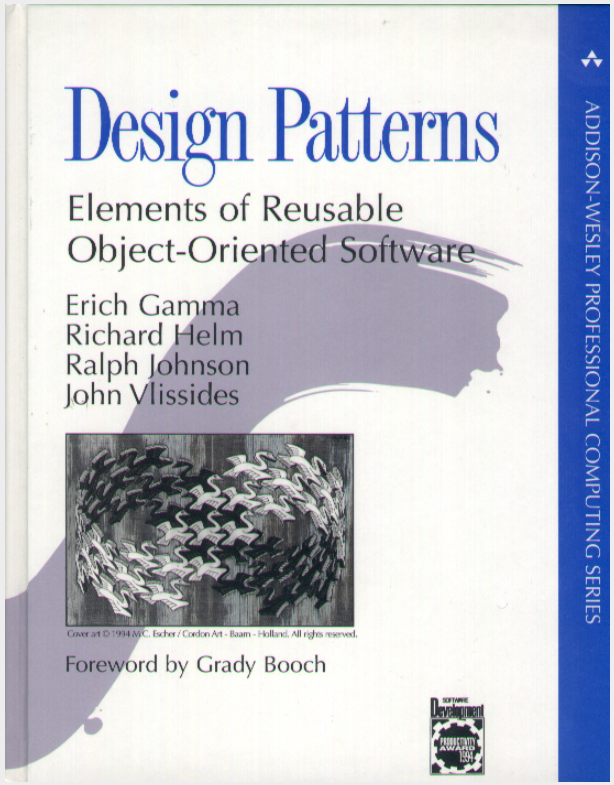

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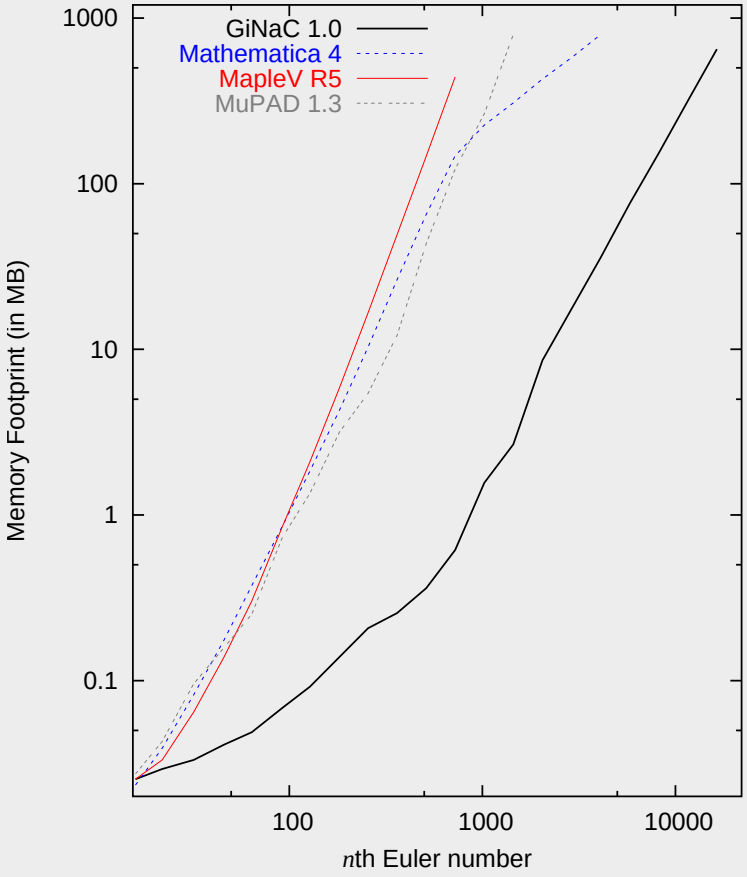
Relevant Design Patterns: Reference Counting

Euler Numbers $E_n :=$ Taylor coefficient in $\frac{1}{\cosh(x)} \equiv \sum_{n=0}^{\infty} E_n \frac{x^n}{n!}$:



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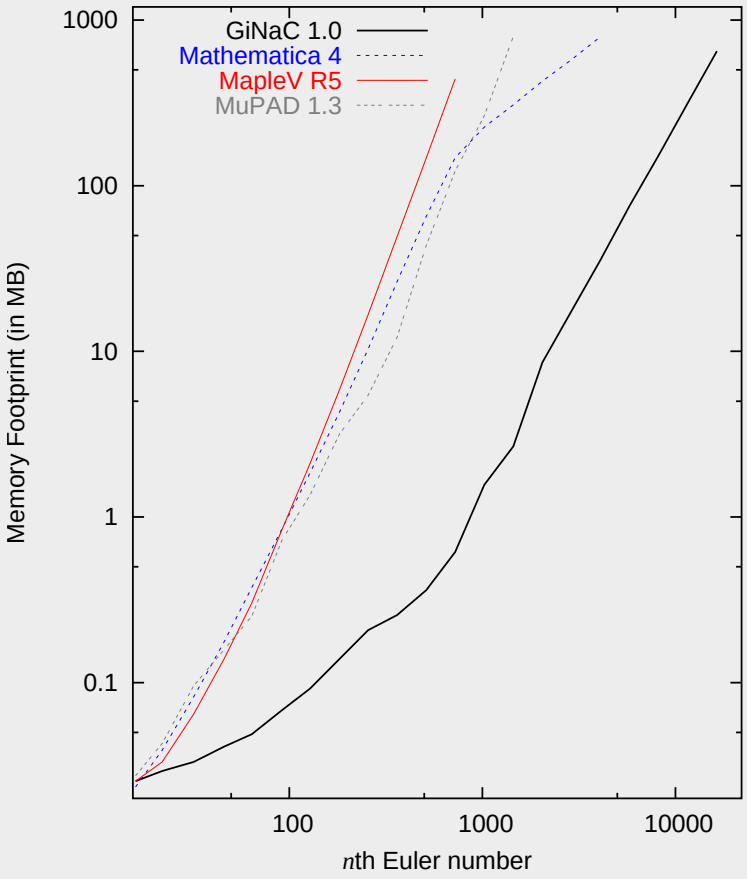
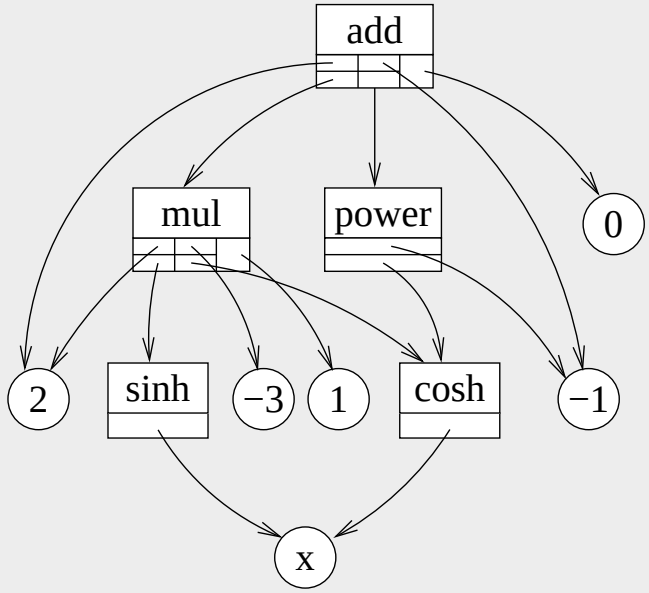


GiNaC's representation tree for

$$\frac{d^2}{dx^2}\frac{1}{\cosh(x)} = 2\frac{\sinh(x)^2}{\cosh(x)^3} - \frac{1}{\cosh(x)}$$

as it appears in E_2 looks like this:



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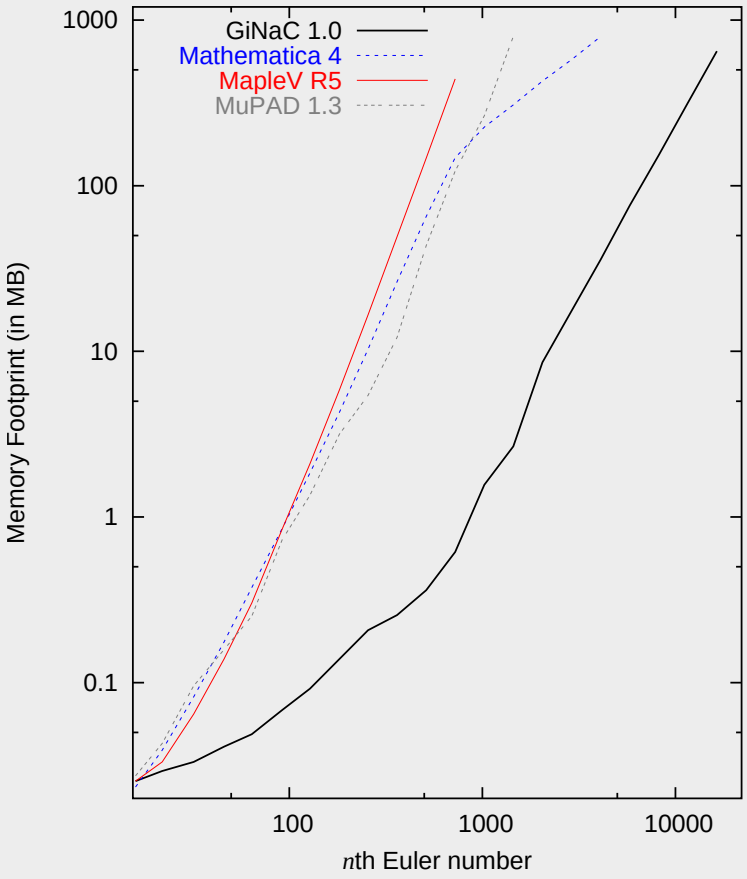

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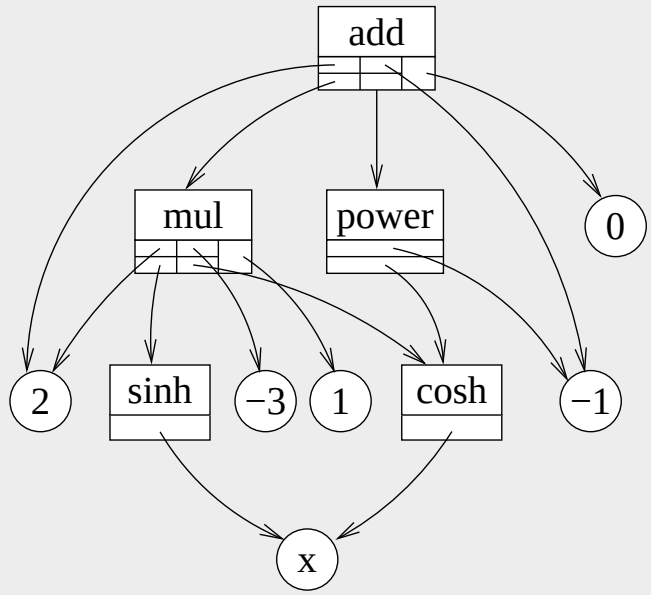

Tree ⇒ DAG

Relevant Design Patterns: Anonymous Evaluation

Problem: Need canonification of symbolic objects (sort of local CSE)

`x-x`	$\longrightarrow$	0	an integer
`x+x`	$\longrightarrow$	`2*x`	a product (or monomial)
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Solution: Implement method `::eval(int)` in each class, call it at the transition between ex and algebraic classes:

```
1  class ex {
2      // ...
3  private:
4      basic* bp;
5  };
```

```
1  ex::ex(const basic& other)
2  {
3      if (!(other.flags & status_flags::evaluated)) {
4          const ex& tmpex = other.eval(1); // evaluate only one (top) level
5          bp = tmpex.bp;
6          ++bp->refcount;
7          // clear up tmpex, etc...
8      }
9      // copy bp, adjust refcount, etc...
10 }
```

Relevant Design Patterns: Delegation vs. the Visitor Pattern

Common algorithms are best implemented as methods using 'Delegation'.

Method is added on each class, wrapper ex uses type dispatch:

```
1  inline const ex
2  ex::expand(void) const
3  {
4      return bp->expand();
5  }
```

```
1  const ex  // virtual
2  basic::expand(void) const
3  {
4      // maybe default...
5  }
```

```
1  const ex
2  add::expand(void) const
3  {
4      // overwrite...
5  }
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etc. . .

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etc. . .

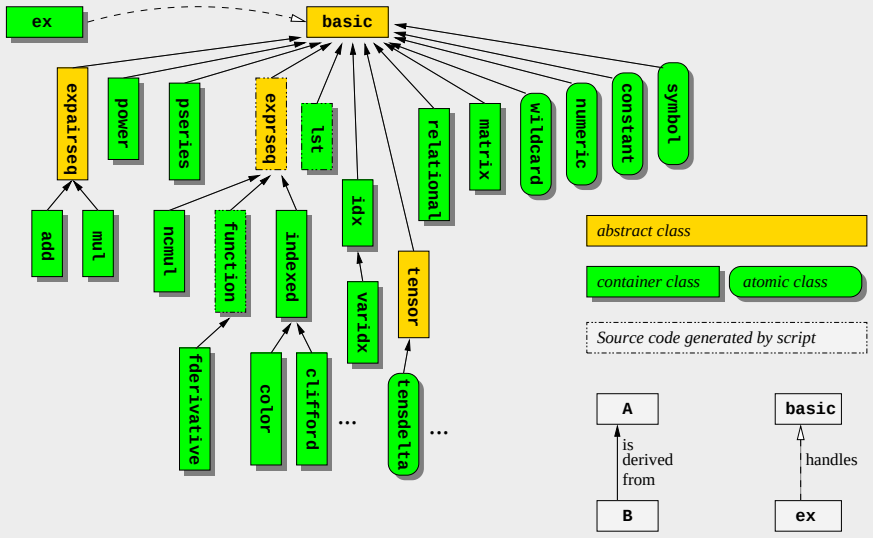
Problem: User-supplied algorithms would require adding methods, recompilation, breaking ABI, etc...
Solution: Let 'Visitor' object traverse the expression tree by reference.

```
struct map_rem_quad
  : public map_function {
    ex var;
    map_rem_quad(const ex& var_)
      : var(var_) {}
    ex operator()(const ex& e)
    {
        if (is_a<add>(e) || is_a<mul>(e))
            return e.map(*this);
        else if (is_a<power>(e))
          /* return zero for even
           * powers of var_ */
        else
            return e;
    }
};

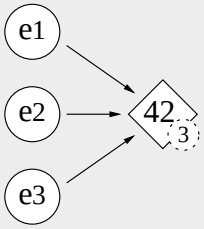
    // ex e constains x...
    map_rem_quad foo(x);
    cout << foo(e) << endl;
```

Relevant Design Patterns: The Flyweight and the Bridge Patterns

Why Flyweights matter:




```
1  ex e1 = 42;
2  ex e2 = e1;
3  ex e3 = 42ULL;
```



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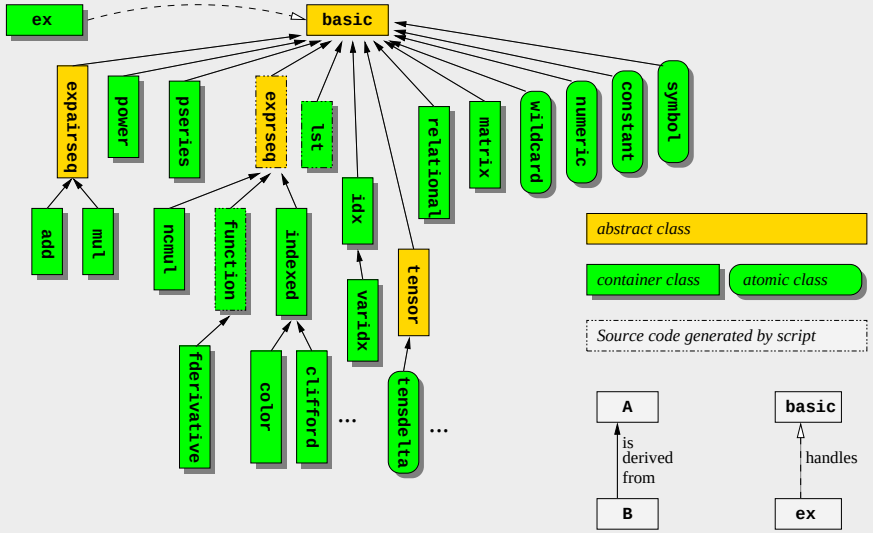


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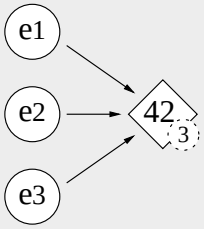

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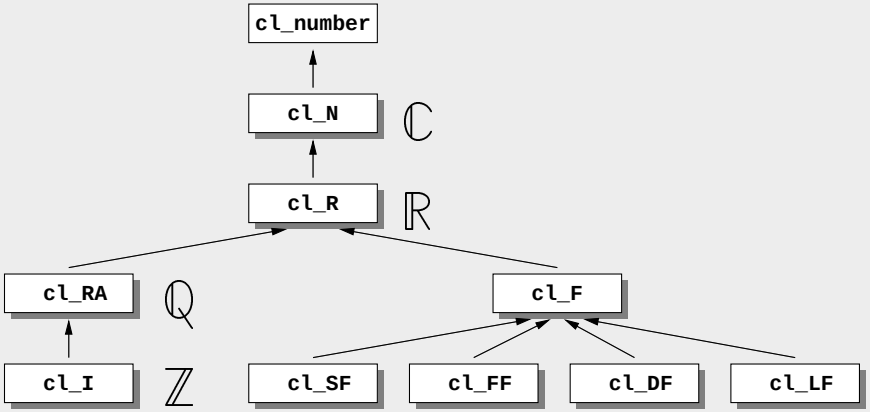
c.f. CLN's class hierarchy:



Positive: mathematically correct

Negative: anti-OO

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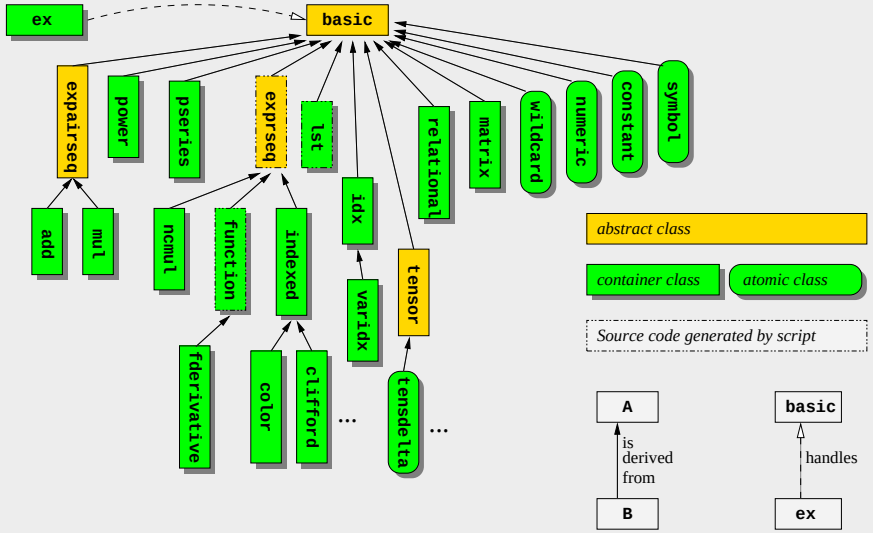


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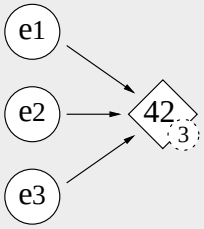

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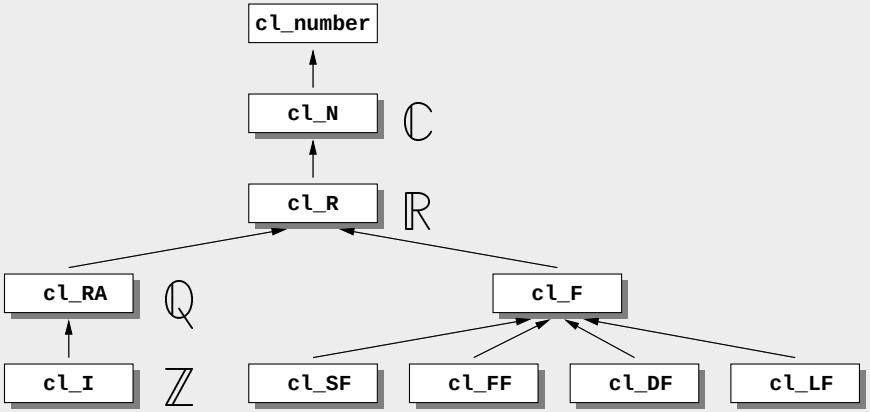

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Solution: implementation must be completely hidden from people `#include`'ing `cl_I`, `cl_RA`, etc. ("Bridge")

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declare e1 to store `x+x`
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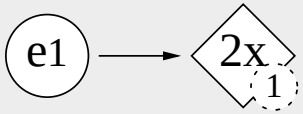


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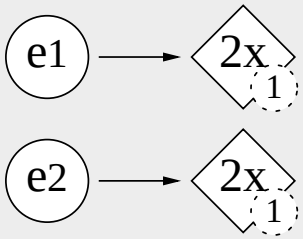

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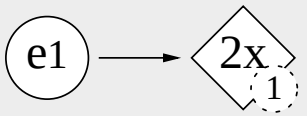


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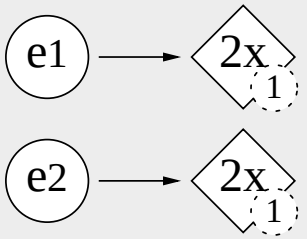


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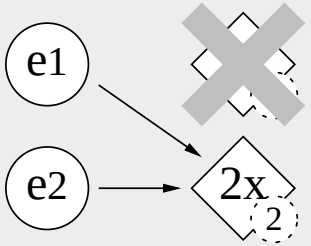

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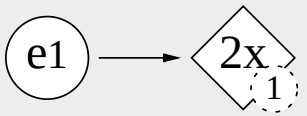


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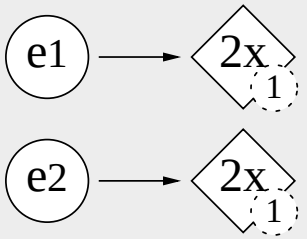


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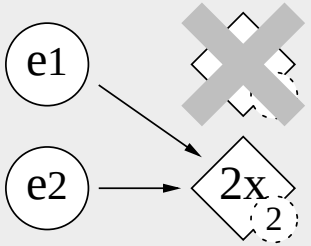

Implementation is straightforward inside `operator==(ex,ex)`!

Prerequisite: Users must not be able to alias to reference-counted objects!
IOW: "Bridge" must be complete.

Availability, spin-offs and all that. . .

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pyginac (Pearu Peterson) Python bindings
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Symbolic Octave (Ben Sapp) Exposes GiNaC to GNU Octave
`http://bsoctave.sourceforge.net/`

Purrs (Roberto Bagnara et al.) Automated complexity analysis
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